

Fringe Patterns of Bose Condensates

S. Métens, D. Lima, P. Borckmans, G. Dewel

Service de Chimie-Physique and

Center for Nonlinear Phenomena and Complex Systems

C.P. 231, Université Libre de Bruxelles,

1050 Bruxelles, Belgium

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We investigate within the Gross-Pitaevski (GP) theory the formation of fringe patterns between two Bose condensates stored in a double well potential modelling the forces applied to the experimental systems. In the case of repulsive interactions between the atoms, we report the onset of interference structures similar to those observed in the experiments, after the release of the confining potential. Conversely, attractive interactions lead to the collapse of the condensate when the number of particles is larger than a critical value. We show that a bias field introduced in the center of the trap allows the system to avoid the blow-up of the density and gives rise to a periodic behavior of growth and decay of spatial modulations reminiscent of the Fermi-Pasta-Ulam recurrence.

The successful experimental realizations [1–4] of Bose-Einstein condensation (BEC) in dilute alkali atomic vapors have sparked a renewed interest in the study of weakly coupled Bose gases. At temperatures close to zero, these magnetically trapped Bose condensed clouds are well described by the condensate wavefunction $\Psi(\mathbf{r}, t)$ which obeys the GP equation initially introduced for superfluid helium [5]

$$i\hbar \frac{\partial \Psi}{\partial t} = \left[-\frac{\hbar^2}{2m} \nabla^2 + U|\Psi|^2 + V_{ext}(\mathbf{r}) \right] \Psi, \quad (1)$$

with the normalization condition

$$\int |\Psi(\mathbf{r}, t)|^2 d\mathbf{r} = N, \quad (2)$$

where N is the number of particles in the condensate. The interactions between atoms of mass m is described by the potential U . Since the thermal wavelength is larger than the characteristic length associated with binary collisions, this potential can simply be characterized by the s -wave scattering length: $U = 4\pi\hbar^2 a/m$. The interaction is repulsive ($a > 0$) for ^{87}Rb and ^{23}Na and attractive ($a < 0$) for ^7Li . The confining potential is given by $V_{ext}(\mathbf{r})$. Many works have been devoted to the characterization of the ground state properties and collective excitations of such condensed gases confined by a single harmonic potential [6–9].

Recently interference fringes have been observed to form when two condensates of repulsive atoms are allowed to expand and overlap [10]. Interference between bose condensates with $a > 0$ has also been the subject of various theoretical studies [11–13]. In this letter, we investigate the formation of such fringe patterns stressing the difference between the cases of attractive and repulsive interaction and we report new dynamical behaviors for bose gases with $a < 0$. In this framework, we numerically solve the GP equation to follow the time evolution of two condensates with an equal number of atoms initially stored in a double well potential. It consists of the superposition of a harmonic potential describing the magnetic trap and a gaussian barrier in the center modelling the laser sheet that cuts the condensate in half. As in the experiments, the iso-potentials of the harmonic trap are ellipsoidal with their long axis defining vertical direction z . The ratio $\Omega = \frac{\omega_z}{\omega_\perp} < 1$ of the corresponding angular frequencies of the trap in the vertical and transverse directions characterizes the anisotropy of the potential. The height of the barrier is chosen to make tunneling ineffective. This initial condition can thus be written in the form:

$$\Psi(z, x, y, 0) = \phi_L(z + d, x, y, 0) + \phi_R(z - d, x, y, 0)e^{i\delta}, \quad (3)$$

and δ is the relative phase which proceeds from the breaking of the groupe symmetry and varies between different experimental runs. The ϕ 's are ground state wavefunctions in each of the two asymmetric traps created by the laser.

We first consider the case of repulsive interaction. When N is sufficiently large (Thomas-Fermi regime), the wavefunctions are then essentially determined by the balance between the particle interaction energy and the external potential as the contribution of the kinetic energy is negligible. The spatial extension L_0 of ϕ is much smaller than the initial separation $2d$ between the two condensates (Fig. 1a). When the double trap is switched off, the condensates expand ballistically and fringe patterns appear in the overlap region. This expansion is essentially dominated by the dispersive effects generated by the kinetic energy term that has been frustrated in the ground state ϕ of the trap. Similar numerical expansion dynamics are indeed obtained for $a = 0$ and $a \neq 0$. The wavelength of these interference structures increases linearly with time $\lambda(t) = \frac{\hbar\pi t}{md}$ and the corresponding expansion velocity $\hbar\pi/md$ decreases for increasing values of the width of the gaussian barrier. The experimental fringe period also becomes smaller for larger powers of the argon laser sheet and thus for larger initial distances between the condensates [10]. The essential dynamics can already be captured by 1D numerical simulations along the z direction that produce the time development illustrated in Fig. 1a which presents strong analogies with the time-resolved diffraction patterns obtained in double-slit atomic experiments [14]. These 1D structures are described by the interference term $I(z, t)$ in the expression of the atomic density $|\psi(\mathbf{r}, t)|^2$ which takes the form:

$$I \propto \cos\left(\frac{2\pi z}{\lambda(t)} + \phi\right). \quad (4)$$

Variation of the relative phase δ only leads to a trivial shift of the whole structure [11]. Taking advantage of the rotational symmetry about the vertical axis, two-dimensional simulations of Eq. (1), that corresponds for instance to a section along the O_{xz} plane of the 3D system produce fringe patterns (Fig. 1b) similar to those seen in the experiments on ^{23}Na [10]. These structures unambiguously demonstrate the coherence of those Bose condensate gas. The system chooses a phase for the macroscopic wavefunction in a process which is analogous to the appearance of coherent oscillations in a laser or a chemical reactor. Indeed the GP equation also corresponds to the strong dispersion limit of the complex Ginzburg-Landau

equation which describes these symmetry breaking bifurcations [15]. The atomic cloud of $5 \cdot 10^5$ atoms then behaves like a single entity to which the de Broglie relation can be applied.

The dynamics is completely different in the case of attractive potentials $U = -|U| < 0$. This arises because the uniform condensate wavefunction $\Psi_0 = \sqrt{n_0}e^{i|U|n_0t/\hbar}$, an exact solution of the (GP) equation in the absence of a confining potential, may undergo a Benjamin-Feir instability [16] (n_0 is the number density in the condensate). Owing to the form of the dispersion relation

$$\omega_k = \frac{\hbar}{2m} \sqrt{k^4 - \frac{4mn_0|U|k^2}{\hbar^2}}, \quad (5)$$

the state Ψ_0 is unstable to long wavelength density perturbations with a wavenumber lying in the range

$$0 < k < \frac{2\sqrt{mn_0|U|}}{\hbar} \quad (6)$$

It had therefore been claimed that BEC of attractive bosons was impossible as this condensation would have corresponded to a mechanically unstable state for which the compressibility would become negative [17]. Nevertheless convincing experimental evidence for BEC in a gas of ^7Li atoms in a trap, consisting of permanent magnets [18], has been reported recently [2]. In such a confined system, BEC can be achieved when the number of particles is sufficiently small to inhibit the destabilizing long wavelength perturbations. More precisely this occurs when $N < N_c = \alpha(l/|a|)$ where l is the typical extension of the ground state in the trap and α is a number of order unity that is determined by the detailed form of the confining potential [6]. When $N > N_c$, the attractive potential overwhelms the zero point energy leading to the collapse of the wavefunction by which, in 2D and 3D, a singularity tends to form in a finite time [19,20]. A sufficient criterion for the collapse (VPT criterion) also yields the critical number N_c . In the collapsing cloud, new effects not contained in Eq. 1 should be taken into account such as various intrinsic inelastic processes which lead to the heating of the sample.

In order to study the interaction between two Bose condensates of attractive atoms,

we consider the same configuration as preceedingly. Here however we maintain some confinement at all times to preserve the integrity of the condensed phase. Initially two stable ($N < N_c$) independent condensates, located near the minima at $z = \pm d$ are separated by the Gaussian barrier. When this barrier is lowered a non-trivial dynamical behavior develops (Fig. 2). During the expansion along the z direction the unstable modes that were excluded from the small initial condensates are awakened by the increase of the size of the system. As a result, a structure appears that invades the whole confining region. Contrary to the case of repulsive interactions, the wavelength does not vary with time anymore and corresponds to that of the fastest growing mode in the unstable range (Eq. (6)). After reaching a maximum value, the amplitude of the modulated density decreases and the system returns to a state of localized condensates near the minima of the potential (Fig. 2). This process repeats periodically and its "superperiod" decreases when the intensity of the barrier is increased. This behavior is reminiscent of the well studied Fermi-Pasta-Ulam (FPU) recurrence and of other recurrences obtained, for instance, with the nonlinear Schrödinger equation [21], that corresponds to the $V_{ext} = 0$ of Eq. (1). It is also analogous to the phenomenon of periodic alternation observed in nonlinear optics experiments [22]. Both phenomena consist of a periodically ordered sequence of quasi-stationary modes. A modal decomposition of the wave function on the most unstable modes reproduces the spatial periodic solutions of the GP equation. The superperiod which can be calculated analytically and expressed in terms of elliptic functions [23] is, as the wavelength, in good agreement with the numerical simulations.

By moving the atoms away from the center along the z axis, the small potential barrier acts to lower the central density hump below the critical value N_c and so prevents the ignition of the collapse of the global system since the cloud can be populated by more atoms before reaching the blow-up. The creation of a vortex in a rotating cloud can lead to a similar effect [8].

At larger values of N (but still $N < N_c$), more unstable modes are allowed, leading to the numerical observation of imperfect recurrences and of more complex behaviors involving

a restricted range of wavenumbers [24]. When N eventually exceeds N_c each condensate collapses separately.

The main effect of the harmonic potential is to confine the system and consequently to reduce the number of unstable modes taking part in the recurrence. When the anisotropy of the confining potential is important, $\Omega < 1$, the fringes are aligned along the transverse direction, and the structure is periodic in the vertical direction. When the system is confined by a weaker potential, transverse modes also fall into the unstable range of wavenumbers (Eq. (5)), leading to cellular structures with spots covering the whole confinement region. They compete with fringes during the recurrent motion, each one taking its turn at dominating the solution profile.

Atomic clouds of near zero temperature, much smaller than the critical condensation temperature, may now be obtained experimentally. In this regime, dissipative effects and thermal fluctuations do not affect the macroscopic dynamics anymore. Furthermore, owing to the presence of the trapping potential, zero temperature fluctuations may be strongly suppressed [25]. Therefore BEC of attractive bosons could provide an ideal candidate to experimentally observe FPU type recurrence phenomena.

In the presence of an initial asymmetric repartition (n_{01} and n_{02}) of the atoms between the two condensates in the double trap potential, we further report numerical observations of an oscillatory exchange of atoms, analogous to the Josephson effect. Numerical simulations in both $1D$ and $2D$ exhibit the already described FPU recurrence (Fig. 3), now accompanied by a periodic exchange of particles locked at half the recurrence frequency. This locking phenomena is observed independently of the initial density repartition.

Contrary to the case of repulsive interactions for which the fringe patterns can be interpreted as resulting from interferences between de Broglie waves, condensates of attractive atoms can exhibit for modulated initial conditions a recurrent behavior that is intimately related to the Benjamin-Feir instability of the uniform macroscopic wavefunction.

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Figure 1. Interference patterns for two freely expanding Bose condensates of ^{23}Na atoms ($a = 4.9 \text{ nm}$, $N = 2 \cdot 10^5$) initially separated by $d = 40 \text{ }\mu\text{m}$ in a double well.

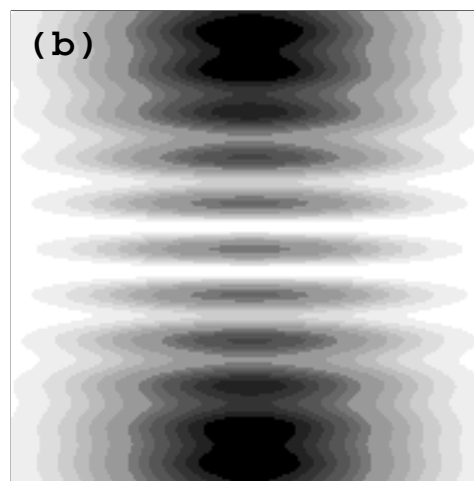
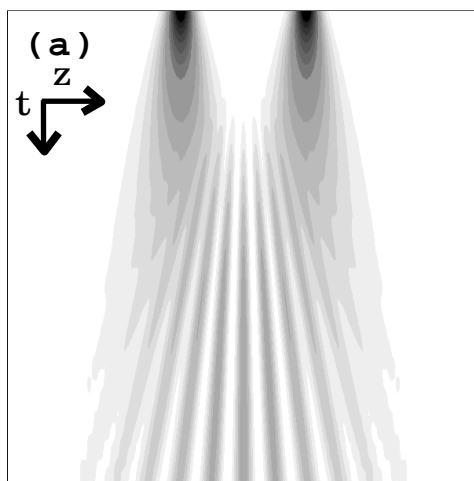
(1a) space-time plot of the density $|\Psi|^2$ obtained by numerically integrating the Gross-Pitaevski equation (Eq. 1) switching off the double-well at $t = 0$. Space is along the z direction and time runs downward up to $t = 40 \text{ ms}$. The linear increase of the wavelength with time is visible.

(1b) density plot in the xz plane after 40 ms when the wavelength equals of $15 \text{ }\mu\text{m}$. The gray scale interpolates between maximum (dark) and minimum (light) densities.

Figure 2. Recurrent behavior of a Bose condensate of ^7Li atoms ($a = -1.45 \text{ nm}$, $N = 700$) evolving in a double well potential: sequence of density plots (plane xz) spanning the recurrence "superperiod". At time $t = 0$, two identical condensates were captured in their

respective well and separated by an adequate barrier. This barrier is lowered to produce the exhibited time evolution.

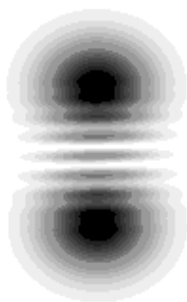
Figure 3. Josephson-type behaviour for a BE condensate of ${}^7\text{Li}$ atoms ($a = -1.45\text{nm}$, $N = 700$). Space-time plot of the density along the z axis during a single exchange of particles along the z axis when the initial transients have died out.



T=0.95



T=1.10



T=1.15



T=1.30



T=1.50



T=1.70



